



Неравенства с модулем

Ответы

1. Неравенства вида «Модуль меньше функции»

1.1. $x \in (-10/3; 4)$. 1.2. $x \in (-6; -3) \cup (-2; 1)$. 1.3. $x \in (-5; -2)$. 1.4. $x \in (-2 + \sqrt{11}; 2\sqrt{7})$.

1.5. $x \in (-1; 3)$. 1.6. $x \in [1 + \sqrt{5}; +\infty)$. 1.7. $x \in (2; 5)$. 1.8. $x = 2$.

2. Неравенства вида «Модуль больше функции»

2.1. $x \in (4/7; +\infty)$. 2.2. $x \in \left(-\infty; \frac{-3 + \sqrt{21}}{2}\right) \cup \left(\frac{-1 + \sqrt{13}}{2}; +\infty\right)$. 2.3. $x \in (-\infty; -1/2] \cup [5; +\infty)$.

2.4. $x \in (-\infty; 1 - \sqrt{10}) \cup (-\sqrt{3}; \sqrt{3}) \cup (1 + \sqrt{10}; +\infty)$. 2.5. $x \in (-\infty; 0] \cup [1; 2] \cup [5; +\infty)$.

3. Неравенства с неотрицательными функциями

3.1. $x \in [-1/3; 3]$. 3.2. $x \in [-2; 11/3]$. 3.3. $x \in (-\infty; -2] \cup [1; +\infty)$. 3.4. $x \in (1 - \sqrt{3}; 1 + \sqrt{3})$.

3.5. $x \in (-\infty; -1) \cup (4; +\infty)$. 3.6. $x \in (-1; 5) \cup (11; +\infty)$. 3.7. $x \in [-3/2; +\infty)$. 3.8.

$x \in [0; 15/4] \cup [4; +\infty)$. 3.9. $x \in (-\infty; -4) \cup (1; 2)$.

4. Общий алгоритм

4.1. $x \in [0; 3]$. 4.2. $x \in [-10; -2]$. 4.3. $x \in [-3; 5]$. 4.4. $x \in [-1; 1]$. 4.5. $x \in (4, 5; +\infty)$.

4.6. $x \in (-\infty; -\sqrt{3}] \cup \{0\} \cup [\sqrt{3}; +\infty)$. 4.7. $(-\infty; -13/5] \cup [3; +\infty)$. 4.8. $x \in [-6; -1] \cup [0; +\infty)$.

4.9. $x \in (-\infty; -2) \cup (2; +\infty)$. 4.10. $x \in (-1 - \sqrt{6}; -1 - \sqrt{2}) \cup (1 + \sqrt{2}; 1 + \sqrt{6})$. 4.11. $x \in (3; 5)$.

4.12. $x \in (-\infty; 1 - \sqrt{5}] \cup [4; +\infty)$.

5. Рациональные дроби

5.1. $x \in (-\infty; -1) \cup (-1; -3/4)$. 5.2. $x \in (-5; -3) \cup (-3; -2) \cup (2; 5)$. 5.3. $x \in (-\infty; -3) \cup (-1; +\infty)$.

5.4. $x \in (-\infty; -5] \cup (-2; 1) \cup [4; +\infty)$. 5.5. $x \in (-2; -1, 5]$. 5.6. $x \in (-3; +\infty)$.

5.7. $x \in [-5; -2) \cup [-1; +\infty)$. 5.8. $x \in [-3; -1)$. 5.9. $x \in (-6; -4) \cup (-4; 1)$.

Решения и комментарии

1. Неравенства вида «Модуль меньше функции»

1.1. $|2x+3| < x+7$

$$|2x+3| < x+7$$

$$|f| < g \Rightarrow -g < f < g$$

$$-x-7 < 2x+3 < x+7$$

$$\begin{cases} 2x+3 < x+7 \\ 2x+3 > -x-7 \end{cases}$$

$$\begin{cases} x < 4 \\ 3x > -10 \\ x > -\frac{10}{3} \end{cases}$$

Омбем: $x \in (-\frac{10}{3}; 4)$

1.2. $|x^2+5x| < 6$

$$|x^2+5x| < 6$$

$$-6 < x^2+5x < 6$$

$$\begin{cases} x^2+5x-6 < 0 \\ x^2+5x+6 > 0 \end{cases} \begin{cases} (x+6)(x-1) < 0 \\ (x+3)(x+2) > 0 \end{cases}$$

Омбем: $x \in (-6; -3) \cup (-2; 1)$

1.3. $|x^2+2x-3| + 3(x+1) < 0$

$$|x^2+2x-3| + 3(x+1) < 0$$

$$|x^2+2x-3| < -3(x+1)$$

$$3x+3 < x^2+2x-3 < -3x-3$$

$$\begin{cases} x^2+5x < 0 \\ x^2-x-6 > 0 \end{cases} \begin{cases} x(x+5) < 0 \\ (x-3)(x+2) > 0 \end{cases}$$

Омбем: $x \in (-5; -2)$

1.4. $|x^2+2x-7| < 2x$

$$|x^2+2x-7| < 2x$$

$$-2x < x^2+2x-7 < 2x$$

$$\begin{cases} x^2-4 < 0 \\ x^2+4x-4 > 0 \end{cases} \begin{cases} x^2-4=0 \\ x^2+4x-4=0 \end{cases}$$

$$\begin{cases} x = \pm 2 \\ x = \frac{-4 \pm \sqrt{16+16}}{2} = -2 \pm \sqrt{4} \end{cases}$$

Омбем: $x \in (-2+\sqrt{4}; 2+\sqrt{4})$

1.5. $|x^2-x-1| < x+2$

$$|x^2-x-1| < x+2$$

$$-x-2 < x^2-x-1 < x+2$$

$$\begin{cases} x^2-2x-3 < 0 \\ x^2+1 > 0 \end{cases}$$

$$(x-3)(x+1) < 0$$

Омбем: $x \in (-1; 3)$

1.6. $|x^2-4x-4| \leq x^2-4$

$$|x^2-4x-4| \leq x^2-4$$

$$4-x^2 \leq x^2-4x-4 \leq x^2-4$$

$$\begin{cases} -4x \leq 0 \Rightarrow x \geq 0 \\ 2x^2-4x-8 \geq 0 \end{cases}$$

$$x^2-2x-4 \geq 0$$

$$D = 4+16=20 \quad x = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$$

Омбем: $x \in [1+\sqrt{5}; +\infty)$

1.7. $|x-3| \cdot |x+1| < 3x-3$

$$|x-3| \cdot |x+1| < 3x-3$$

$$|x-3| \cdot |x+1| = |(x-3)(x+1)| = |x^2-2x-3|$$

$$|x^2-2x-3| < 3x-3$$

$$3-3x < x^2-2x-3 < 3x-3$$

$$\begin{cases} x^2-5x < 0 \\ x^2+x-6 > 0 \end{cases} \begin{cases} x(x-5) < 0 \\ (x+3)(x-2) > 0 \end{cases}$$

Омбем: $x \in (2; 5)$

1.8. $|x^3-2x-4| \leq 2x-4$

$$|x^3-2x-4| \leq 2x-4$$

$$4-2x \leq x^3-2x-4 \leq 2x-4$$

$$\begin{cases} x^3-4x \leq 0 \\ x^3-8 \geq 0 \end{cases} \begin{cases} x(x-2)(x+2) \leq 0 \\ x \geq 2 \end{cases}$$

Омбем: $x \in [2; 2)$

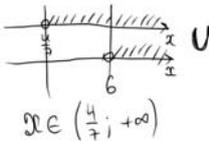
2. Неравенства вида «Модуль больше функции»

2.1. $|3x+1| > 5-4x$

$$|3x+1| > 5-4x$$

$$\begin{cases} 3x+1 > 5-4x \\ 3x+1 < -5+4x \end{cases}$$

$$\begin{cases} 7x > 4 & x > \frac{4}{7} \\ x > 6 & x > 6 \end{cases}$$



2.3. $|2x^2-9x+15| \geq 20$

$$|2x^2-9x+15| \geq 20$$

$$\begin{cases} 2x^2-9x+15 \geq 20 \\ 2x^2-9x+15 \leq -20 \end{cases}$$

$$\begin{cases} 2x^2-9x-5 \geq 0 \\ 2x^2-9x+35 \leq 0 \end{cases}$$

$$\begin{cases} 2x^2-9x-5 \geq 0 & 2x^2-9x+35 \leq 0 \\ D=81+40=121 & D=81-140 < 0 \end{cases}$$

$$x = \frac{9 \pm 11}{4} \rightarrow \frac{5}{2} \quad \frac{-1}{2} \quad 5 \quad x$$

Ответ: $x \in (-\infty; -\frac{1}{2}] \cup [\frac{5}{2}; +\infty)$

2.2. $|x^2+2x-3| > x$

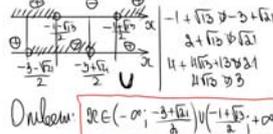
$$|x^2+2x-3| > x$$

$$\begin{cases} x^2+2x-3 > x \\ x^2+2x-3 < -x \end{cases}$$

$$\begin{cases} x^2+x-3 > 0 \\ x^2+3x-3 < 0 \end{cases}$$

$$\begin{cases} x^2+x-3=0 & x^2+3x-3=0 \\ D_1=1+12=13 & D_2=9+12=21 \end{cases}$$

$$x = \frac{-1 \pm \sqrt{13}}{2} \quad x = \frac{-3 \pm \sqrt{21}}{2}$$



2.4. $|x^2-x-6| > x+3$

$$|x^2-x-6| > x+3$$

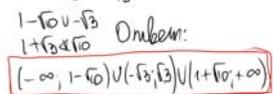
$$\begin{cases} x^2-x-6 > x+3 \\ x^2-x-6 < -x-3 \end{cases}$$

$$\begin{cases} x^2-2x-9 > 0 \\ x^2-3 < 0 \end{cases}$$

$$\begin{cases} x^2-2x-9 > 0 & x^2-3 < 0 \end{cases}$$

$$D=4+36=40 \quad x = \pm\sqrt{10}$$

$$x = \frac{2 \pm 2\sqrt{10}}{2} = 1 \pm \sqrt{10}$$



2.5. $||x^2-8x+2|-x^2| \geq 2x+2$

$$||x^2-8x+2|-x^2| \geq 2x+2$$

$$\begin{cases} |x^2-8x+2|-x^2 \geq 2x+2 \\ |x^2-8x+2|-x^2 \leq -2x-2 \end{cases}$$

$$\begin{cases} x^2-8x+2 \geq x^2+2x+2 \\ x^2-8x+2 \leq -x^2-2x-2 \end{cases}$$

$$\begin{cases} -8x \geq 2x & 2x^2-6x+4 \leq 0 \\ x \leq -\frac{1}{2} & x^2-3x+2 \leq 0 \end{cases}$$

$$\begin{cases} x \leq -\frac{1}{2} & (x-1)(x-2) \leq 0 \end{cases}$$

$$\begin{cases} x \leq -\frac{1}{2} & x \in [1; 2] \end{cases}$$

Ответ: $x \in (-\infty; 0] \cup [1; 2] \cup [5; +\infty)$

3. Неравенства с неотрицательными функциями

3.1. $|x+2| \geq |1-2x|$

$$|x+2| \geq |1-2x|$$

$$|x+2| \geq |1-2x| \rightarrow \begin{cases} x+2 \geq 1-2x \\ x+2 \leq -1+2x \end{cases}$$

$$\begin{cases} 3x \geq -1 & x \geq -\frac{1}{3} \\ x+2 \leq -1+2x & x \geq 3 \end{cases}$$

$$\begin{cases} x \geq -\frac{1}{3} \\ x \geq 3 \end{cases}$$

Ответ: $x \in [-\frac{1}{3}; 3]$

3.2. $|13-2x| \geq |4x-9|$

$$|13-2x| \geq |4x-9|$$

$$(13-2x)^2 \geq (4x-9)^2$$

$$(13-2x)^2 - (4x-9)^2 \leq 0$$

$$(13-2x-4x+9)(13-2x+4x-9) \leq 0$$

$$(22-6x)(4+2x-9) \leq 0$$

$$(22-6x)(2x-5) \leq 0$$

$$\begin{cases} 22-6x \geq 0 & x \leq \frac{11}{3} \\ 2x-5 \geq 0 & x \geq \frac{5}{2} \end{cases}$$

Ответ: $x \in [\frac{5}{2}; \frac{11}{3}]$

3.3. $3|x+1| \geq |x+5|$

$$\begin{aligned} 3|x+1| &\geq |x+5| \\ (3x+3)^2 &\geq (x+5)^2 \\ (3x+3-x-5)(3x+3+x+5) &\geq 0 \\ (2x-2)(4x+8) &\geq 0 \\ (x-1)(x+2) &\geq 0 \end{aligned}$$

Ombem: $x \in (-\infty; -2] \cup [1; +\infty)$

3.5. $|3x+5| < x^2+1$

$$\begin{aligned} |3x+5| < x^2+1 &\Rightarrow (3x+5)^2 < (x^2+1)^2 \\ (x^2+1-3x-5)(x^2+1+3x+5) &> 0 \\ (x^2-3x-4)(x^2+3x+6) &> 0 \\ x^2-3x-4 > 0 & \quad x^2+3x+6 > 0 \end{aligned}$$

Ombem: $x \in (-\infty; -1) \cup (4; +\infty)$

3.7. $|x^2+x+1| \leq |x^2+3x+4|$

$$\begin{aligned} |x^2+x+1| &\leq |x^2+3x+4| \\ (x^2+x+1)^2 &\leq (x^2+3x+4)^2 \\ ((x^2+x+1) - (x^2+3x+4)) \times & \\ &\times ((x^2+x+1) + (x^2+3x+4)) \leq 0 \\ (-2x-3)(2x^2+4x+5) &\leq 0 \\ x = -\frac{3}{2} \quad D = 16-40 < 0 \quad \phi & \end{aligned}$$

Ombem: $x \in [-\frac{3}{2}; +\infty)$

3.9. $|x^2-3x-3| > |x^2+7x-13|$

$$\begin{aligned} |x^2-3x-3| &> |x^2+7x-13| \\ (x^2-3x-3)^2 &> (x^2+7x-13)^2 \\ (x^2-3x-3)^2 - (x^2+7x-13)^2 &> 0 \\ (x^2-3x-3-x^2-7x+13) \times & \\ &\times (x^2-3x-3+x^2+7x-13) > 0 \\ (10-10x)(2x^2+4x-16) &> 0 \\ (x-1)(x^2+2x-8) &< 0 \\ (x-1)(x+4)(x-2) &< 0 \end{aligned}$$

Ombem: $x \in (-\infty; -4) \cup (1; 2)$

3.4. $x^2 < 2|x+1|$

$$\begin{aligned} x^2 < 2|x+1| &\Rightarrow (x^2)^2 < (2x+2)^2 \\ (x^2-2x-2)(x^2+2x+2) &< 0 \\ x^2-2x-2 < 0 & \quad > 0 \\ D = 4+8 = 12 \quad x = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3} & \end{aligned}$$

Ombem: $x \in (1-\sqrt{3}; 1+\sqrt{3})$

3.6. $|x-3| \cdot |x-2| < |x^2-3x-16|$

$$\begin{aligned} |x-3| \cdot |x-2| &< |x^2-3x-16| \\ |x-3| \cdot |x-2| = |(x-3)(x-2)| = |x^2-5x+6| & \\ |x^2-5x+6| &< |x^2-3x-16| \\ (x^2-5x+6)^2 &< (x^2-3x-16)^2 \\ (x^2-5x+6-x^2+3x+16) \times & \\ &\times (x^2-5x+6+x^2-3x-16) < 0 \\ (-2x+22)(2x^2-8x-10) &< 0 \\ (x-11)(x^2-4x-5) &> 0 \\ (x-11)(x-5)(x+1) &> 0 \end{aligned}$$

Ombem: $x \in (-\infty; -1) \cup (5; 11)$

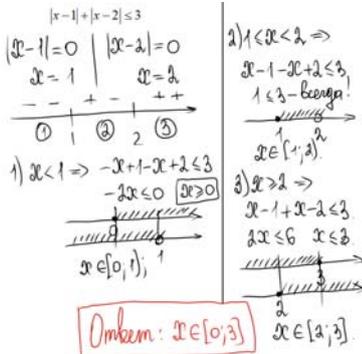
3.8. $|x^2-8x+15| \leq |15-x^2|$

$$\begin{aligned} |x^2-8x+15| &\leq |15-x^2| \\ (x^2-8x+15)^2 &\leq (15-x^2)^2 \\ (x^2-8x+15)^2 - (15-x^2)^2 &\leq 0 \\ (x^2-8x+15-x^2+15) \times & \\ &\times (x^2-8x+15+x^2-15) \leq 0 \\ (30-8x)(2x^2-8x) &\leq 0 \quad x(x-4)(4x-15) \geq 0 \end{aligned}$$

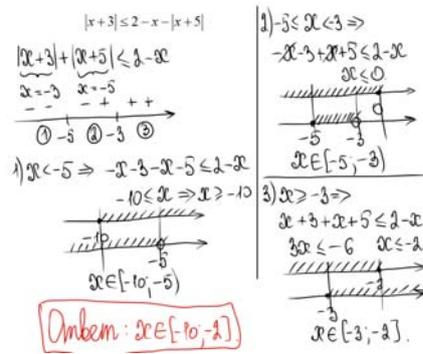
Ombem: $x \in [0; \frac{15}{4}] \cup [4; +\infty)$

4. Общий алгоритм

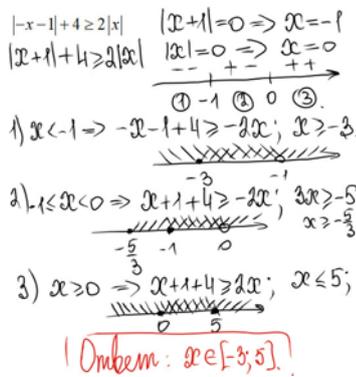
4.1. $|x-1| + |x-2| \leq 3$



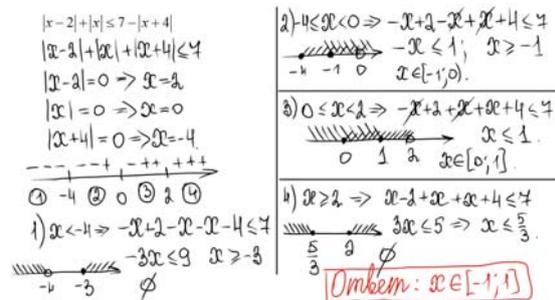
4.2. $|x+3| \leq 2-x-|x+5|$



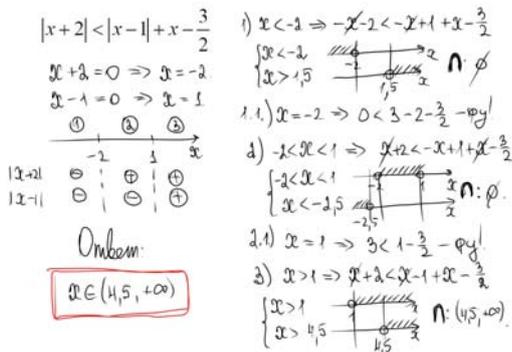
4.3. $|-x-1| + 4 \geq 2|x|$



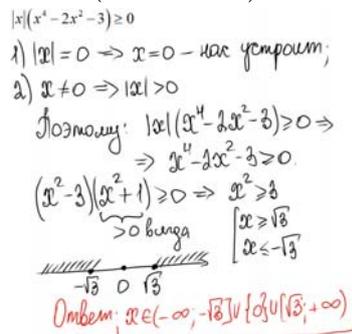
4.4. $|x-2| + |x| \leq 7-|x+4|$



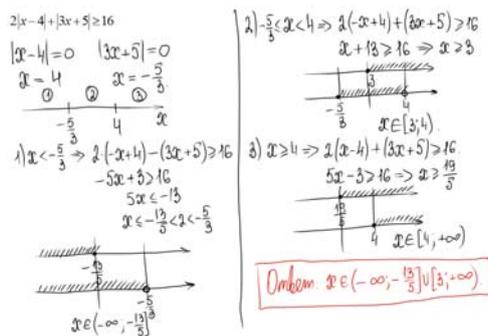
4.5. $|x+2| < |x-1| + x - \frac{3}{2}$



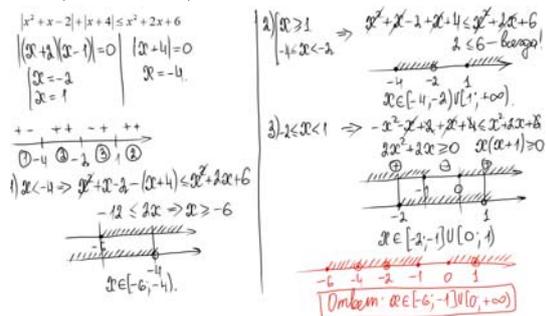
4.6. $|x|(x^4 - 2x^2 - 3) \geq 0$



4.7. $2|x-4| + |3x+5| \geq 16$



4.8. $|x^2 + x - 2| + |x+4| \leq x^2 + 2x + 6$



4.9. $x^2 - |x| > 2$

$x^2 - |x| > 2$
 $x^2 = |x| \Rightarrow |x|^2 - |x| > 2$
 $|x| = t \Rightarrow t^2 - t - 2 > 0$
 $(t-2)(t+1) > 0$

$t < -1 \Rightarrow |x| < -1 \Rightarrow \emptyset$
 $t > 2 \Rightarrow |x| > 2 \Rightarrow \begin{cases} x > 2 \\ x < -2 \end{cases}$
 Ответ: $x \in (-\infty; -2) \cup (2; +\infty)$

4.11. $2|x| - |x^2 - 4x| > x$

$2|x| - |x^2 - 4x| > x$ | $|x| = 0 \Rightarrow x = 0$
 $- + + - + + \rightarrow |x^2 - 4x| = 0 \Rightarrow \begin{cases} x = 0 \\ x = 4 \end{cases}$

1) $-2x - x^2 + 4x > x$
 $-x(x-3) > 0$
 $x(x-3) < 0$
 $x \in (3; 4)$

2) $2x + x^2 - 4x > x$
 $x(x-3) > 0$
 $x \in (0; 3) \cup (4; +\infty)$

3) $2x - x^2 + 4x > x$
 $-x(x-5) > 0$
 $x \in (4; 5)$

Ответ: $x \in (3; 5)$

4.10. $|x^2 - 2|x| - 3| < 2$

$|x^2 - 2|x| - 3| < 2$
 $x^2 = |x| \Rightarrow |x| = t \Rightarrow |t^2 - 2t - 3| < 2$
 $-2 < t^2 - 2t - 3 < 2 \Rightarrow \begin{cases} t^2 - 2t - 5 < 0 \\ t^2 - 2t - 1 > 0 \end{cases}$

$t^2 - 2t - 5 = 0$ | $t^2 - 2t - 1 = 0$
 $D = 4 + 20 = 24$ | $D = 4 + 4 = 8$;
 $t = \frac{2 \pm \sqrt{24}}{2} = 1 \pm \sqrt{6}$ | $t = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$

1) $-1 < t < 1 - \sqrt{2} < 0 \Rightarrow \emptyset$
 1) $1 + \sqrt{2} < t < 1 + \sqrt{6} \Rightarrow 0(x)$
 Ответ: $x \in (-1 - \sqrt{6}; -1 - \sqrt{2}) \cup (1 + \sqrt{2}; 1 + \sqrt{6})$

4.12. $|x^2 - 3x + 2| - |2 - x| \geq 4$

$|x^2 - 3x + 2| - |2 - x| \geq 4$ | $x^2 - 3x + 2 = 0$ | $x - 2 = 0$
 $(x-2)(x-1) \geq 4 + |x-2|$ | $x = 2$; $x = 1$ | $x = 2$

1) $x < 1 \Rightarrow x^2 - 3x + 2 \geq 4 - x + 2$
 $x^2 - 2x - 4 \geq 0$
 $D = 4 + 16 = 20 \Rightarrow x = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$
 $x \in (-\infty; 1 - \sqrt{5}]$

2) $1 \leq x < 2 \Rightarrow -x^2 + 3x - 2 \geq 4 - x + 2$
 $x^2 - 4x + 8 \leq 0$
 $D = 16 - 32 < 0 \Rightarrow \emptyset$

3) $x \geq 2 \Rightarrow x^2 - 3x + 2 \geq 4 + x - 2$
 $x^2 - 4x \geq 0$ | $x(x-4) \geq 0$
 $x \in [4; +\infty)$

Ответ: $x \in (-\infty; 1 - \sqrt{5}] \cup [4; +\infty)$

5. Рациональные дроби

5.1. $\left| \frac{2x+1}{x+1} \right| > 2$

$\left| \frac{2x+1}{x+1} \right| > 2$
 $\frac{2x+1}{x+1} > 2$ | $\frac{2x+1}{x+1} < -2$
 $\frac{2x+1}{x+1} - 2 > 0$ | $\frac{2x+1}{x+1} + 2 < 0$

Ответ: $x \in (-\infty; -1) \cup (-1; -3/4)$

5.3. $\frac{|x+3|}{|x+2|-1} \geq 1$

$\frac{|x+3|}{|x+2|-1} \geq 1$ | $|x+3| - |x+2| + 1 \geq 0$

1) $x < -3 \Rightarrow \frac{-x-3+x+2+1}{-x-2-1} \geq 0$
 $\frac{-x-3+x+2+1}{-x-2-1} \geq 0$

2) $x < -2 \Rightarrow \frac{x+3-x-2+1}{-x-2-1} \geq 0$
 $\frac{x+3-x-2+1}{-x-2-1} \geq 0$
 $\frac{x+3-x-2+1}{-x-2-1} \geq 0$
 $\frac{x+3-x-2+1}{-x-2-1} \geq 0$
 $\frac{x+3-x-2+1}{-x-2-1} \geq 0$

3) $x > -2 \Rightarrow \frac{x+3-x-2+1}{x+2-1} \geq 0$
 $\frac{x+3-x-2+1}{x+2-1} \geq 0$
 $\frac{x+3-x-2+1}{x+2-1} \geq 0$
 $\frac{x+3-x-2+1}{x+2-1} \geq 0$
 $\frac{x+3-x-2+1}{x+2-1} \geq 0$

Ответ: $x \in (-\infty; -3) \cup (-2; -1) \cup (0; +\infty)$

5.2. $\frac{x^2 - 7|x| + 10}{x^2 + 6x + 9} < 0$

$\frac{x^2 - 7|x| + 10}{x^2 + 6x + 9} < 0$
 $x^2 - 7|x| + 10 = 0$ | $x^2 + 6x + 9 = 0$
 $|x|^2 - 7|x| + 10 = 0$ | $(x+3)^2 = 0$
 $(x-2)(x-5) = 0$ | $x = -3$ (3x)
 $|x| = 2 \Rightarrow x = \pm 2$ | $x = -3$
 $|x| = 5 \Rightarrow x = \pm 5$ | $x = -3$

Ответ: $x \in (-5; -3) \cup (-3; 2) \cup (2; 5)$

5.4. $\frac{|1+2x|}{x^2 + x - 2} \leq \frac{1}{2}$

$\frac{|1+2x|}{x^2 + x - 2} \leq \frac{1}{2}$ | $x^2 + x - 2 > 0 \Rightarrow |1+2x| \leq x^2 + x - 2$
 $-x^2 - x + 2 \leq |1+2x| \leq x^2 + x - 2$
 $|1+2x| < 1$ | $(x^2 - 3x - 1) > 0$ | $(x-1)(x+1) \geq 0$
 $|1+2x| < 1$ | $x^2 + 5x \geq 0$ | $x(x+5) \geq 0$
 $|1+2x| < 1$ | $x^2 + x - 2 > 0$ | $(x+2)(x-1) > 0$

Ответ: $x \in (-\infty; -5) \cup (-2; 1) \cup (4; +\infty)$

$$5.5. \frac{|x+3|}{x^2+5x+6} \geq 2.$$

$$\frac{|x+3|}{x^2+5x+6} \geq 2$$

$$x^2+5x+6 > 0$$

$$(x+3)(x+2) > 0$$

$$|x+3|=0$$

$$x=-3$$

$$x < -3 \quad \frac{-(x+3)}{(x+3)(x+2)} \geq 2$$

$$\boxed{x+2 < -1} \quad \frac{-1}{x+2} \geq 2 \quad | \cdot (x+2) < 0$$

$$-1 \leq 2x+4 \quad \emptyset$$

$$2x \geq -5 \Rightarrow x \geq -2.5$$

$$x > -2 \quad \frac{1}{x+2} \geq 2 \quad | \cdot (x+2) > 0$$

$$\boxed{x+2 > 0} \quad 1 \geq 2x+4 \quad 2x \leq -3$$

$$-2 < x \leq -1.5$$

Омбем: $x \in (-2; -1.5]$

$$5.7.: \frac{|x+3|+x}{x+2} \geq 1.$$

$$\frac{|x+3|+x}{x+2} \geq 1$$

$$|x+3|-2 \geq 0$$

$$x+2 \neq 0 \quad x \neq -2$$

$$|x+3|=2 \quad \begin{cases} x+3=2 & x=-1 \\ x+3=-2 & x=-5 \end{cases}$$

Омбем: $x \in [-5; -2) \cup [-1; +\infty)$

$$5.9. \frac{x^2+5x-6}{|x+4|} < 0$$

$$\frac{x^2+5x-6}{|x+4|} < 0$$

O.D.3: $|x+4| \neq 0 \Rightarrow x \neq -4$
 Ho $x \neq -4 \Rightarrow |x+4| > 0$

Изоморфизм: $\frac{(x+6)(x-1)}{|x+4|} < 0 \Rightarrow$

Омбем: $x \in (-6; -4) \cup (-4; 1)$

$$5.6. \frac{x^2-|x|-12}{x+3} \leq 2x.$$

$$\frac{x^2-|x|-12}{x+3} \leq 2x$$

$$\frac{(x+3)(x-4)}{x+3} \leq 2x$$

$$1) x < 0 \Rightarrow \frac{x^2+x-12-2x(x+3)}{x+3} \leq 0$$

$$\frac{-3x^2-5x-12}{x+3} \leq 0 \quad \frac{x^2+5x+12}{x+3} \geq 0$$

$$x^2+5x+12 > 0 \Rightarrow x+3 > 0 \Rightarrow x > -3.$$

$$1.1) x=0 \Rightarrow -4 \leq 0 - 0 \cdot (-)$$

$$2) x > 0 \Rightarrow x-4 \leq 2x \Rightarrow x \geq -4$$

Омбем: $x \in (-3; +\infty)$

$$5.8. \frac{x^2+4x+3}{|x+1|} \leq 0$$

$$\frac{x^2+4x+3}{|x+1|} \leq 0$$

O.D.3: $|x+1| \neq 0 \Rightarrow x \neq -1$
 Ho ecum $x \neq -1$, mo $|x+1| > 0$

Изоморфизм: $\frac{(x+3)(x+1)}{|x+1|} \leq 0 \Rightarrow \begin{cases} (x+3)(x+1) \leq 0 \\ x \neq -1 \end{cases}$

Омбем: $x \in [-3; -1)$